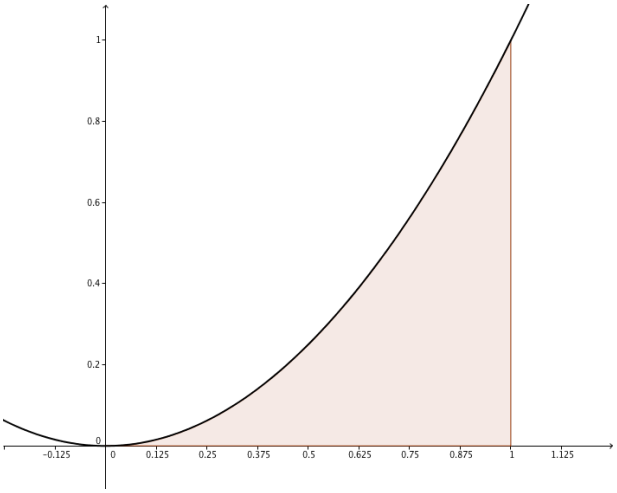
The Area and Distance Problems

The Area Problem

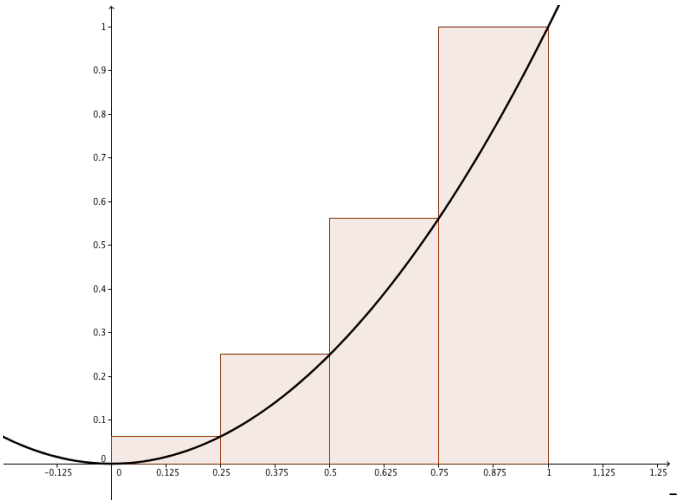
Find the area of the region S that lies under the curve $y = f(x)$ from a to b. In this lecture we will try to find the area of the region that lies under the curve $y = x^2$ from 0 to 1.

- **Idea**



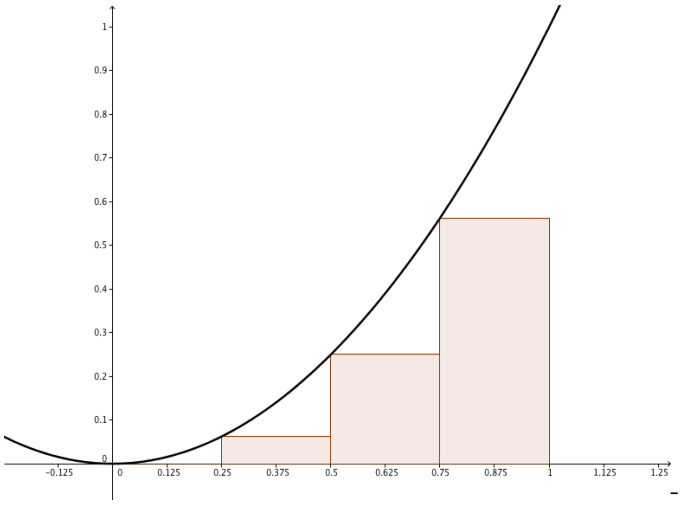
Example 1.

Approximate the area under the graph of $f(x) = x^2$ from 0 to 1 by using $n = 4$ right endpoint rectangles of equal width.



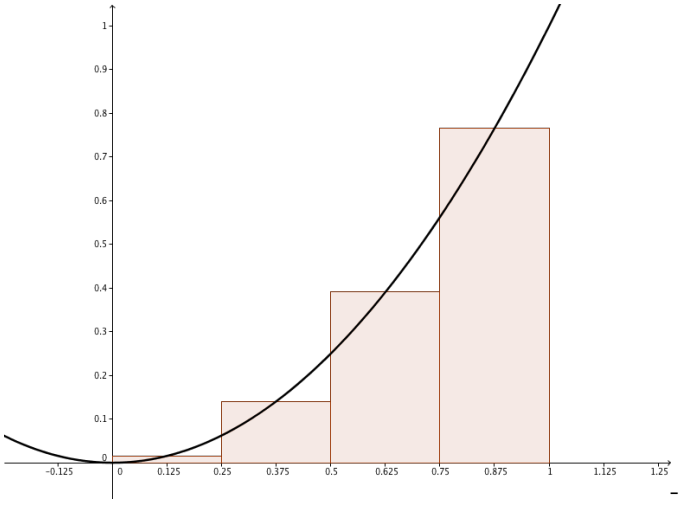
Example 2.

Approximate the area under the graph of $f(x) = x^2$ from 0 to 1 by using $n = 4$ left endpoint rectangles of equal width.



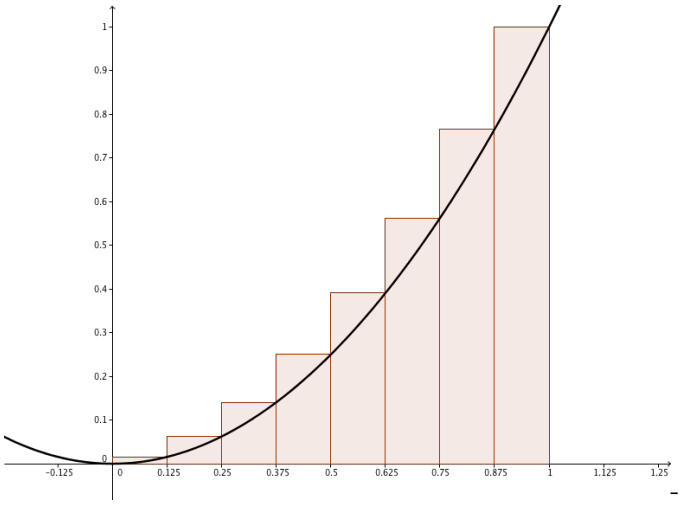
Example 3.

Approximate the area under the graph of $f(x) = x^2$ from 0 to 1 by using $n = 4$ midpoint rectangles of equal width.

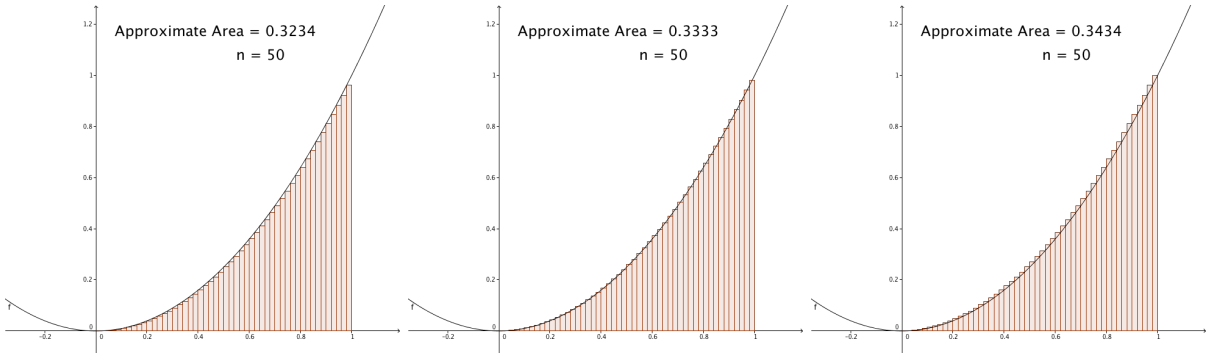


Example 4.

Approximate the area under the graph of $f(x) = x^2$ from 0 to 1 by using $n = 8$ right endpoint rectangles of equal width.



What happens if we use more rectangles?



Example 5.

Approximate the area under the graph of $f(x) = x^2$ from 0 to 1 by using an arbitrary n number of right endpoint rectangles of equal width.

Example 6.

Compute the limit of the approximation in Example 5 as we let $n \to \infty$.

The Area Under a Curve

The **area** A of the region S that lies under the graph of the continuous function f from $x = a$ to $x = b$ is the limit of the sum of the areas of approximating rectangles

$$A = \lim_{n\to\infty} [f(x_1^*)\Delta x + f(x_2^*)\Delta x + \cdots + f(x_n^*)\Delta x]$$

The Distance Problem

Find the distance traveled by an object during a certain time period if the velocity of the object is known at all times.

- **Idea**

Example 7.

We want to estimate the distance our car drives during a 30-second time interval. We take speedometer readings every five seconds and record them in the following table:

Time (s)	0	5	10	15	20	25	30
Velocity (ft/s)	25	31	35	43	47	46	41