Antiderivatives and Indefinite Integrals

Introduction

Antiderivatives

Antiderivatives

A function F is an **antiderivative** of a function f on an interval I if

$$F'(x) = f(x)$$

for all x in I.

Theorem

If F is an antiderivative of f on interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

Theorem

If F and G are differentiable functions and $F'(x) = G'(x)$ for all x in some interval then $G(x) = F(x) + C$

Example 1.

Find the most general antiderivatives of each of the following functions:

1. $f(x) = \sin x$

2. $f(x) = \frac{1}{x}$

3. $f(x) = x^n \qquad n \neq -1$

Indefinite Integrals

Table of Indefinite Integrals

$$\int k\,dx = kx + C$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C \ (n \neq -1) \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C \qquad \int a^x\,dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C \qquad \int \csc x \cot x\,dx = -\csc x + C$$

$$\int \frac{1}{1+x^2}\,dx = \arctan x + C \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$$

Properties of Indefinite Integrals

Constant	$\int k \cdot f(x)\,dx = k \cdot \int f(x)\,dx$
Sum	$\int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$
Difference	$\int f(x) - g(x)\,dx = \int f(x)\,dx - \int g(x)\,dx$

Example 2.

1. $\int 5\ dx$

2. $\int 9e^x\ dx$

3. $\int (4x^3 + 2x - 1)\ dx$

4. $\int \left(10x^4 - 2\sec^2 x\right)\ dx$

Example 3.

1. $\displaystyle\int \frac{x^3 - 3}{x^2}\, dx$

2. $\displaystyle\int \left(\frac{2}{\sqrt[3]{x}} - 6\sqrt{x} \right)\, dx$

3. $\displaystyle\int x(x^2 + 2)\, dx$

4. $\displaystyle\int \frac{\cos\theta}{\sin^2\theta}\, d\theta$

Example 4.

1. Find f if $f'(x) = e^x + 20(1 + x^2)^{-1}$ and $f(0) = -2$

2. Find f if $f''(x) = 12x^2 + 6x - 4$ and $f(0) = 4$ and $f(1) = 1$

Example 5.

The graph of a function f is given below. Make a rough sketch of an antiderivative F, given that $F(0) = 2$