

Curve Sketching

Graphing Strategy

1. **Analyze $f(x)$.**
 - (a) Domain of f
 - (b) Intercepts.
 - (c) Symmetry
 - (d) Asymptotes.
2. **Analyze $f'(x)$.**
 - Determine the intervals on which f is increasing and decreasing
 - Find local maxima and minima of f .
3. **Analyze $f''(x)$.**
 - Determine the intervals on which the graph of f is concave upward and downward.
 - Find the inflection points of f .
4. **Sketch the graph of f .**
 - (a) Draw asymptotes, intercepts, local maxima and minima, and inflection points.
 - (b) Sketch in what you know from steps 1-3
 - (c) Plot additional points as needed and complete the sketch.

Domain

Intercepts

Symmetry

Asymptotes

Example 1.

Sketch a possible graph of a function f that satisfies the following conditions:

1. First Derivative:

(a) $f'(x) > 0$ on $(-\infty, 1)$

(b) $f'(x) < 0$ on $(1, \infty)$.

2. Second Derivative:

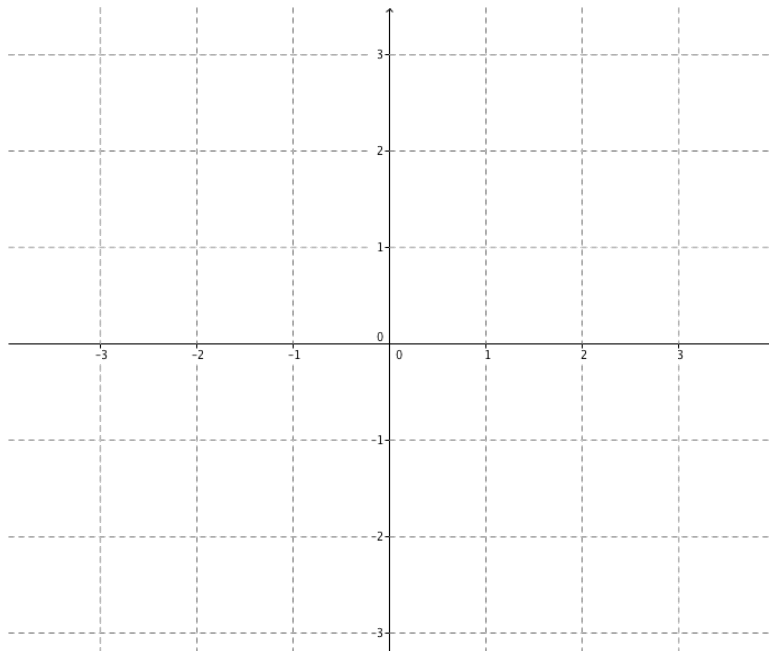
(a) $f''(x) > 0$ on $(-\infty, -2)$ and $(2, \infty)$

(b) $f''(x) < 0$ on $(-2, 2)$.

3. Long-Term Behavior:

(a) $\lim_{x \rightarrow -\infty} f(x) = -2$

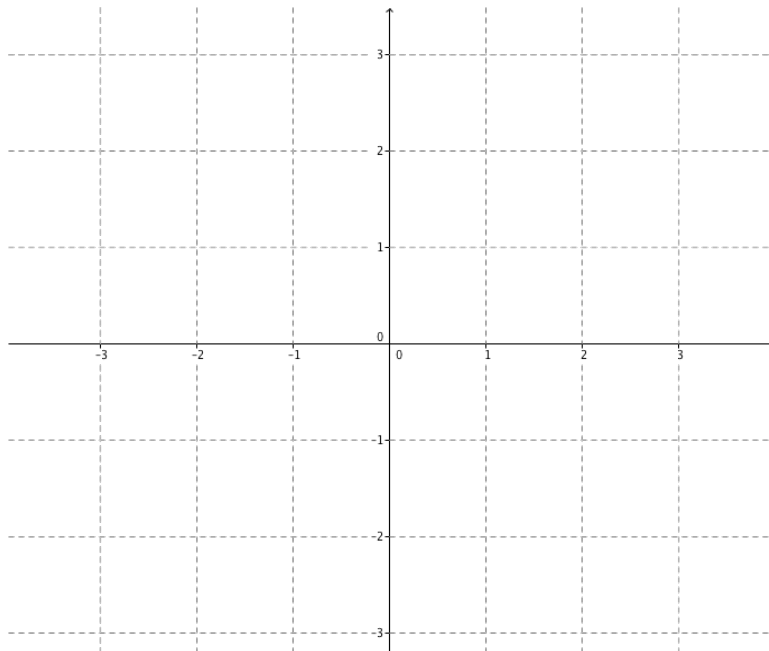
(b) $\lim_{x \rightarrow \infty} f(x) = 0$



Example 2.

Sketch the graph of the function

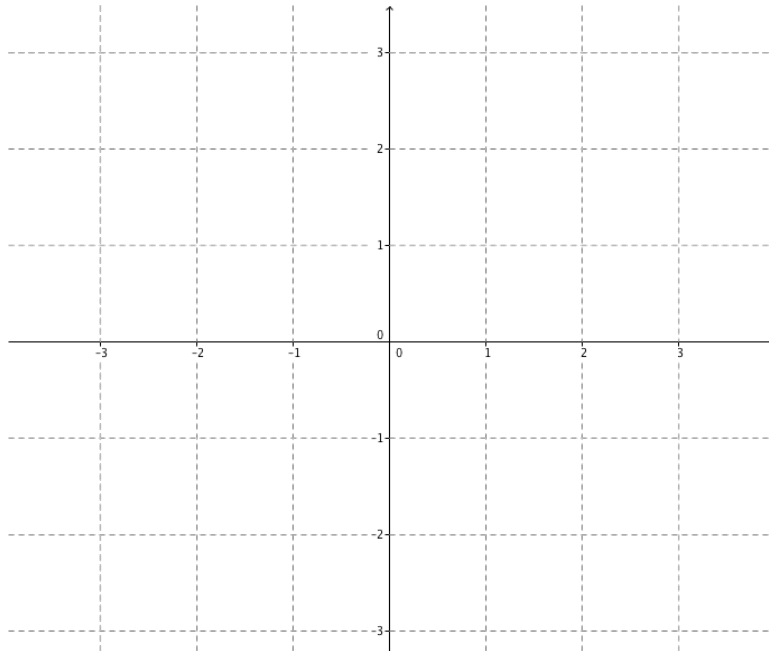
$$f(x) = \frac{2x^2}{x^2 - 1}$$



Example 3.

Sketch the graph of the function

$$g(x) = \frac{x^2}{\sqrt{x+1}}$$



Example 4.

Sketch the graph of the function

$$f(x) = xe^x$$

