

## Derivatives of Logarithmic Functions

We apply the implicit differentiation technique to differentiate logarithmic functions.

### Derivative of Logarithmic Function

$$\frac{d}{dx} (\log_a x) = \frac{1}{x \ln a}$$

### Derivative of Natural Logarithmic Function

$$\frac{d}{dx} (\ln x) = \frac{1}{x}$$

### General Logarithmic Rule

If  $u(x)$  is a differentiable function, and

$$y = f(x) = \ln(u(x))$$

then

$$f'(x) = \frac{1}{u(x)}u'(x)$$

#### Example 1.

Differentiate  $y = \ln(x^3 + 1)$ .

**Example 2.**

Find  $\frac{d}{dx} (\ln(\sin x))$ .

**Example 3.**

Differentiate  $f(x) = \sqrt{\ln x}$ .

**Example 4.**

Differentiate  $f(x) = \log_{10}(2 + \sin x)$ .

**Example 5.**

Find  $\frac{d}{dx} \left( \ln \left( \frac{x+1}{\sqrt{x-2}} \right) \right)$ .

**Example 6.**

Find  $f'(x)$  if  $f(x) = \ln |x|$ .

## The Number $e$ .

### Another Definition of $e$

$$e = \lim_{x \rightarrow 0} (1 + x)^{1/x}$$