Derivatives of Polynomials

In this lecture we establish some basic differentiation rules that allow us to compute the derivative of power functions and polynomials.

Derivative Notation

If $y = f(x)$, then

$$f'(x) \qquad y' \qquad \frac{dy}{dx} \qquad \frac{d}{dx}\left[f(x)\right]$$

all represent the derivative of f at x.

Power Functions

Derivative of a Constant Function

$$\frac{d}{dx}(c) = 0$$

Derivative of the Identity Function

$$\frac{d}{dx}(x) = 1$$

The Power Rule (General Version)

If n is any real number, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Example 1.

Differentiate:

1. $f(x) = x^6$

2. $y = x^{1000}$

3. $y = t^4$

4. $\dfrac{d}{dr}(r^3)$

Example 2. [Rewrite function first]

Differentiate:

1. $f(x) = \dfrac{1}{x^2}$

2. $y = \sqrt[3]{x^2}$

Example 3.

Find equations of the tangent line and normal line to the curve $y = x\sqrt{x}$ at the point $(1, 1)$.

- **normal line:**

General Differentiation Properties

Three Differentiation Properties

If c is a constant and f and g are differentiable functions, then

Constant Multiple Rule
$$\frac{d}{dx}\left[c \cdot f(x)\right] = c \cdot \frac{d}{dx}\left[f(x)\right]$$

Sum Rule
$$\frac{d}{dx}\left[f(x) + g(x)\right] = \frac{d}{dx}\left[f(x)\right] + \frac{d}{dx}\left[g(x)\right]$$

Difference Rule
$$\frac{d}{dx}\left[f(x) - g(x)\right] = \frac{d}{dx}\left[f(x)\right] - \frac{d}{dx}\left[g(x)\right]$$

Example 4 [Constant Multiple Rule]

1. $\dfrac{d}{dx}\left[3x^4\right]$

2. $\dfrac{d}{dx}\left[-x\right]$

Example 5 [Sum/Difference Rule]

1. $\dfrac{d}{dx}\left[x^4 + x^2 - x\right]$

2. $\dfrac{d}{dx}\left[x^8 + 12x^5 - 4x^4 + 10x^3 - 6x + 5\right]$

Example 6

Find the points on the curve $y = x^4 - 6x^2 + 4$ where the tangent line is horizontal.

Example 7

The equation of motion of a particle is $s(t) = 2t^3 - 5t^2 + 3t + 4$, where s is measured in centimeters and t in seconds. Find the acceleration as a function of time. What is the acceleration after 2 seconds?