Exponential Growth and Decay

Introduction to Differential Equations.

A differential equation...

$$\frac{dy}{dt} = ky$$

Theorem

The only solutions of the differential equation $\frac{dy}{dt} = ky$ are the exponential functions

$$y(t) = y(0)e^{kt}$$

Population Growth

Example 1.

Use the fact that the world population was 2560 million in 1950 and 3040 million in 1960 to model the population of the world in the second half of the 20th century. (Assume that the growth rate is proportional to the population size.)

1. What is the relative growth rate?

2. Use the model to estimate the world population in 1993 and to predict the population in the year 2020.

Radioactive Decay

Physicists express the rate of decay in terms of **half-time**, the time required for half of any given quantity to decay.

Example 2.

The half-life of radium-226 is 1590 years.

1. A sample of radium-226 has a mass of 100 mg. Find a formula for the mass of the sample that remains after t years.

2. Find the mass after 1000 years correct to the nearest milligram.

3. When will the mass be reduced to 30 mg?

Newton's Law of Cooling

The rate of cooling (or warming) of an object is proportional to the temperature difference between the object and its surroundings, provided that the difference is not too large.

$$\frac{dT}{dt} = k(T - T_s)$$

Example 3.

A bottle of soda pop at room temperature ($72^\circ F$) is placed in a refrigerator where the temperature is $44^\circ F$. After half an hour the soda pop has cooled to $61^\circ F$.

1. What is the temperature of the soda pop after another half hour?

2. How long does it take for the soda pop to cool to $50^\circ F$?