The First Derivative and the Shape of the Graph

In this section our goal is to deduce facts about a function from information about its first derivative.

The Shape of a Graph

Increasing/Decreasing Test

For the interval (a, b), if $f' > 0$, then f is increasing, and if $f' < 0$, then f is decreasing.

$f'(x)$	$f(x)$	Graph of f	Examples

Finding Intervals of Increase or Decrease

As the previous theorem indicates finding the intervals of increase or decrease of a function f is equivalent to finding the intervals on which its first derivative f' is positive or negative.

Partition Numbers of f'

A **partition number** of the function f' is a real number c such that either

1. $f'(c) = 0$ or
2. $f'(c)$ DNE

Example 1.

Find the intervals on which the function $f(x) = 1 + x^3$ is increasing or decreasing.

Example 2.

Find the intervals on which the function $f(x) = (1 - x)^{1/3}$ is increasing or decreasing.

Example 3.

Find the intervals on which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is increasing or decreasing.

Finding Local Maximum and Minimum Values

Critical Numbers

- A **critical number** of a function f is a real number c in the domain of f such that either
  1. $f'(c) = 0$ or
  2. $f'(c)$ DNE
- A **critical point** of a function f is the point $(c, f(c))$ where c is a critical number.

Fermat's Theorem

If $f(c)$ is a local extremum of the function f, then c is a critical number of f.

Strategy:

First Derivative Test

Let c be a critical number of f.

- If $f'(x)$ changes from negative to positive at c, then $f(c)$ is a local minimum.
- If $f'(x)$ changes from positive to negative at c, then $f(c)$ is a local maximum.
- If $f'(x)$ does not change sign at c, then $f(c)$ is not a local extremum.

Example 4.

Find the local minimum and maximum values of the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$.

Example 5.

Find the local minimum and maximum values of the function

$$g(x) = x + 2\sin x \qquad 0 \leq x \leq 2\pi$$

Example 6.

Find the local minimum and maximum values of the function

$$f(x) = \frac{1}{(x-2)^2}$$

Finding Absolute Maximum and Minimum Values

We have already seen how to use the first derivative and the **Closed Interval Method** to find the absolute extreme values for a function defined on a closed interval. Under special circumstances we can use the first derivative to find an absolute extreme value for a function defined on an open interval.

First Derivative Test for Absolute Extreme Values

Let c be the **only** critical number of f.

- If $f'(x)$ changes from negative to positive at c, then $f(c)$ is the absolute minimum value of f.
- If $f'(x)$ changes from positive to negative at c, then $f(c)$ is the absolute maximum value of f.

Example 7.

Find any absolute extreme values of the function

$$A(x) = 2400 - 2x^2$$