Implicit Differentiation

Introduction

As we have seen, equations are often used to define a function. Until now we have only considered the differentiation of functions defined by equations of the form: $y = f(x)$.

Ways we can use an equation to define a function:

- Explicitly

- Implicitly

We can use the technique of Implicit Differentiation to calculate the derivative of an implicitly defined function.

Implicit Differentiation

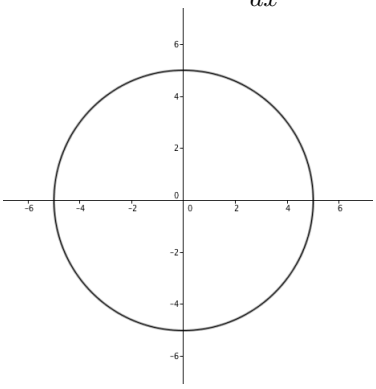
1. Differentiate both sides of the equation with respect to x.
2. Use Chain Rule to differentiate all y terms.
3. Solve the resulting equation for y'.

Example 1.

If $3x^2 + y - 2 = 0$ find y'.

Example 2.

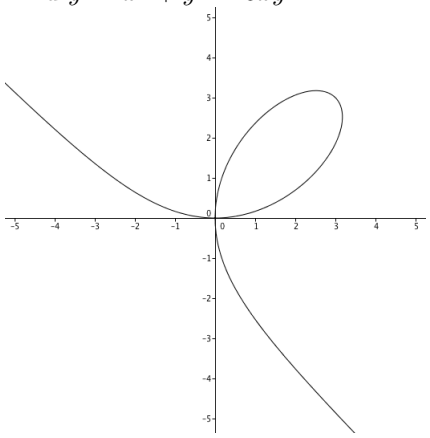
1. If $x^2 + y^2 = 25$, find $\dfrac{dy}{dx}$.



2. Find an equation of the tangent to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$.

Example 3.

1. Find y' if $x^3 + y^3 = 6xy$



2. Find the tangent to the folium of Descartes $x^3 + y^3 = 6xy$ at the point $(3, 3)$.

3. At what point in the first quadrant is the tangent line horizontal?

Example 4.

Find y' if $\sin(x+y) = y^2 \cos x$.

Example 5.

Find y'' if $x^4 + y^4 = 16$.