Introduction

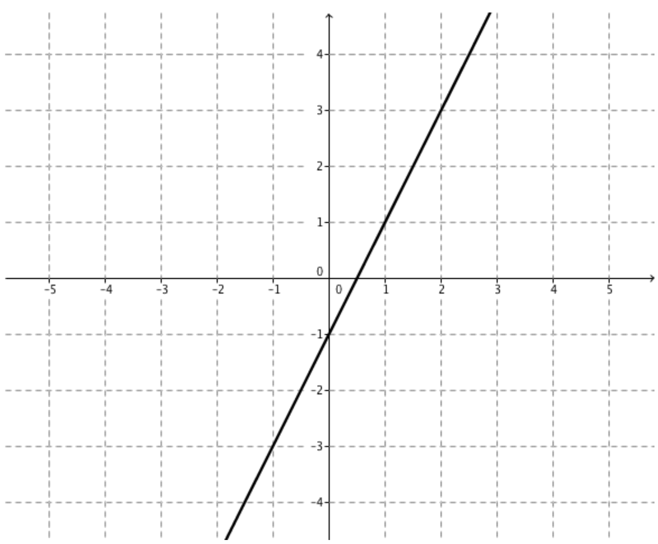
Definition of a Function

Function

A **function** is a correspondence between two sets of elements such that to each element in the first set, there corresponds one and only one element in the second set.

- the first set is called the **domain**.
- the second set is called the **codomain**.
- the subset of corresponding elements in the second set is called the **range**.

Although there are many ways to specify a function we will almost exclusively be using equations and graphs.



Domain and Range from a Graph

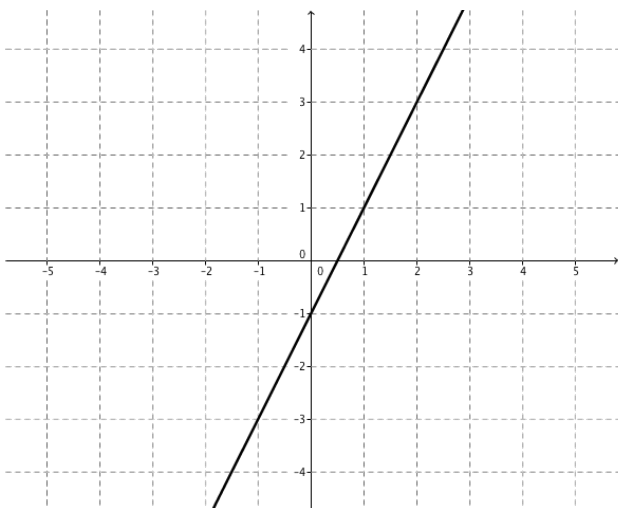
Domain. If a vertical line at $x = a$ intersects the graph at only one point then a is in the domain.

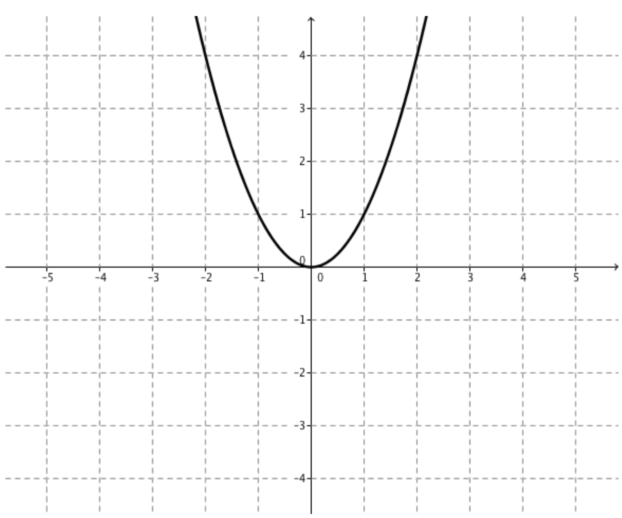
Vertical Line Test If a vertical line at $x = a$ intersects the graph at more than one point then the graph is not a function.

Range. If a horizontal line at $y = b$ intersects the graph at one or more points then b is in the range.

Example 1.

Find the domain and range of the following functions.





Example 2.

Evaluate the following for:

$$f(x) = \frac{12}{x-2} \qquad g(x) = 1 - x^2 \qquad h(x) = \sqrt{x-1}$$

1. $g(0)$

2. $f(2)$

3. $h(-3)$

Example 3.

Evaluate the following for

$$f(x) = x^2 - 2x + 7 \qquad h \neq 0$$

1. $f(a)$

2. $f(a+h)$

3. $\frac{f(a+h)-f(a)}{h}$

Catalog of Common Functions

- Polynomials.
- Power Functions.
- Rational Functions.
- Algebraic Functions.
- Trigonometric Functions.
- Piecewise Functions
- Exponential Functions.
- Logarithmic Functions.
- Inverse Trigonometric Functions.

Finding the Domain of a Function.

Once we know that an equation does specify a function we need to determine its domain.

1. Start with domain $D = \mathbb{R} = (-\infty, \infty)$.
2. Remove any x values for which the function is undefined:
   - Any x values that give a 0 in the denominator
   - Any x values that give a negative under an even root sign.
   - Any x values that give a negative or 0 inside the log or ln sign.

Piecewise Functions

Functions that are defined by different formulas in different regions of their domains.

Example 4.

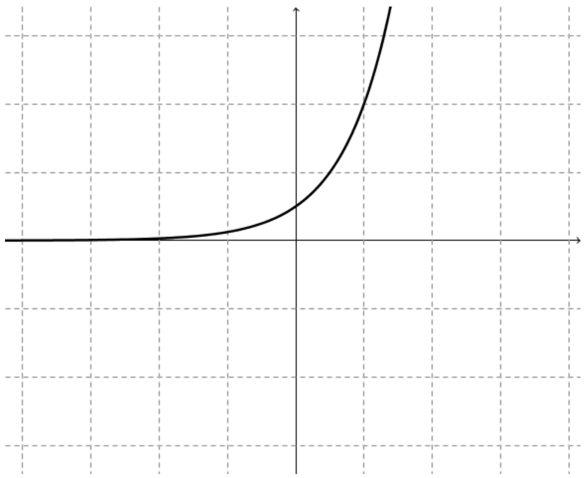
1. Consider the absolute value function

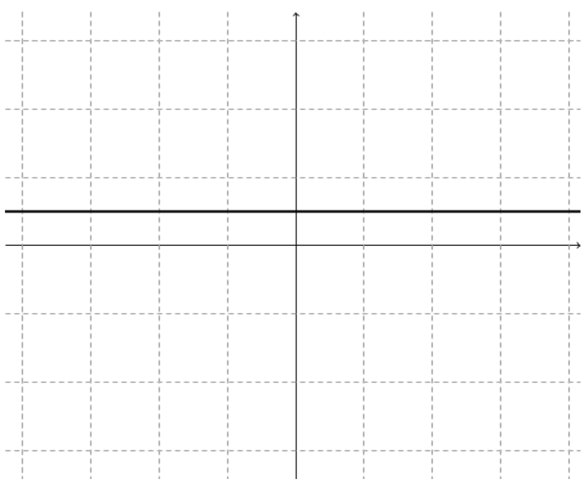
$$|x| = \begin{cases} -x & \textbf{if } x < 0 \\ x & \textbf{if } x \geq 0 \end{cases}$$

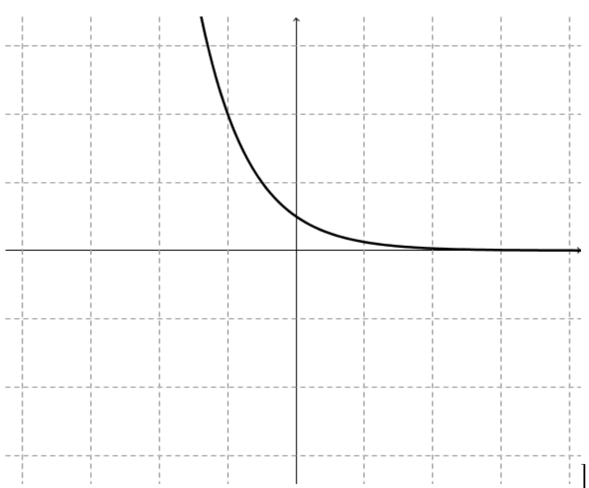
2. Consider the function

$$f(x) = \begin{cases} 1 - x & \textbf{if } x < -1 \\ x^2 & \textbf{if } x > -1 \end{cases}$$

Exponential Functions







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Example 5.

1. Sketch the function $y = e^{-x} - 1$ and state the domain and range.

2. Find the domain of the function
$$f(x) = \frac{1}{e^{-x} - 1}$$

Law of Exponents

If a and b are positive numbers and x and y are any real numbers, then

1. $a^{x+y} = a^x a^y$
2. $a^{x-y} = \frac{a^x}{a^y}$
3. $(a^x)^y = a^{xy}$
4. $(ab)^x = a^x b^x$

Example 6.

Use the Law of Exponents to simplify the expression.

1. $4^{-3}/2^{-8}$
2. $1/\sqrt[3]{x^4}$

Inverse Functions

Inverse Function

Given a function f with domain A and range B, its **inverse function** f^{-1} has domain B and range A and is defined by

$$f^{-1}(y) = x \qquad \iff \qquad f(x) = y$$

for any y in B.

Cancellation Properties

Example 7.

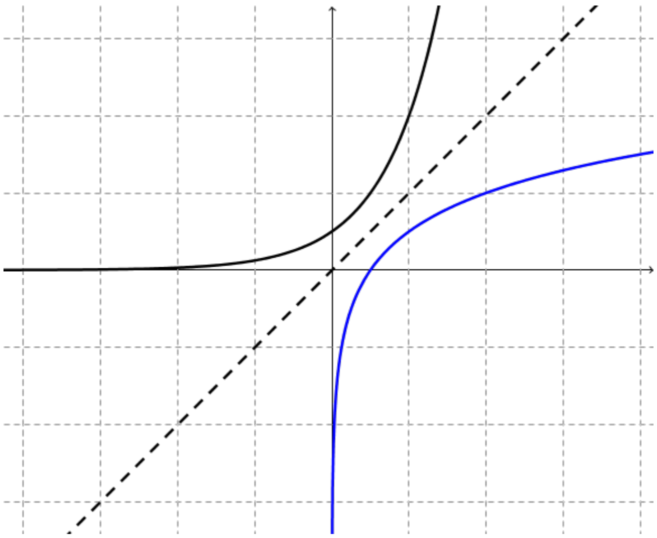
Verify the cancellation properties for the functions

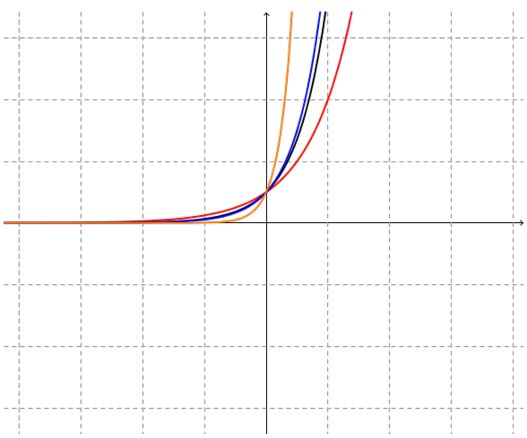
$$f(x) = x^3 + 2 \qquad f^{-1}(x) = \sqrt[3]{x-2}$$

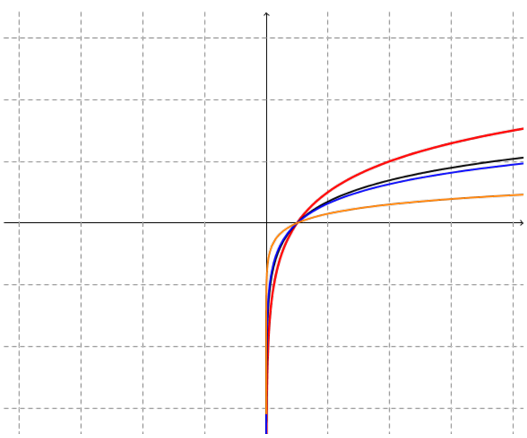
The graph of an inverse function f^{-1} is found by reflecting the graph of the original function f about the line $y = x$.

Logarithmic Functions

Given an exponential function $f(x) = a^x$ we can find its inverse function $f^{-1}(x) = \log_a x$ by reflecting its graph across the line $y = x$.







Laws of Logarithms

If x and y are positive numbers and r is any real number, then

1. $\log_a(xy) = \log_a x + \log_a y$
2. $\log_a(\frac{x}{y}) = \log_a x - \log_a y$
3. $\log_a x^r = r \log_a x$

Example 8.

1. Use the laws of logarithms to evaluate $\log_2 80 - \log_2 5$

2. Express $\ln a + .5 \ln b$ as a single logarithm.

Example 9.

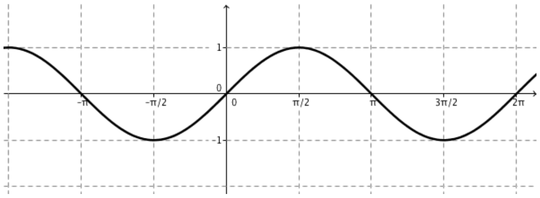
1. Find x if $\ln x = 5$.

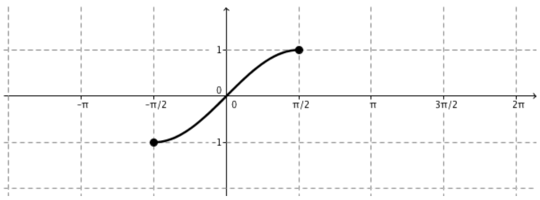
2. Solve the equation $e^{5-3x} = 10$.

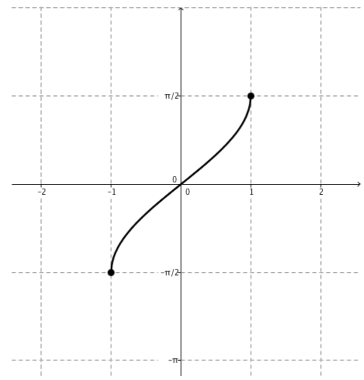
Example 10.

Sketch the graph of the function $y = \ln(x - 2) - 1$.

Inverse Trigonometric Functions







Inverse Sine Function

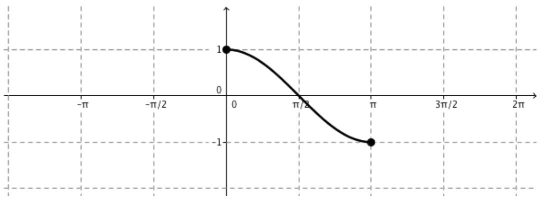
$$\sin^{-1} x = y \qquad \Longleftrightarrow \qquad \sin y = x \qquad \text{and} \qquad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

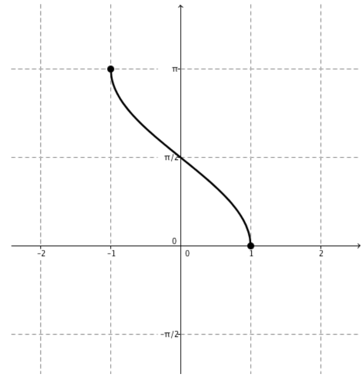
Example 11.

Evaluate the following:

1. $\sin^{-1}(1/2)$

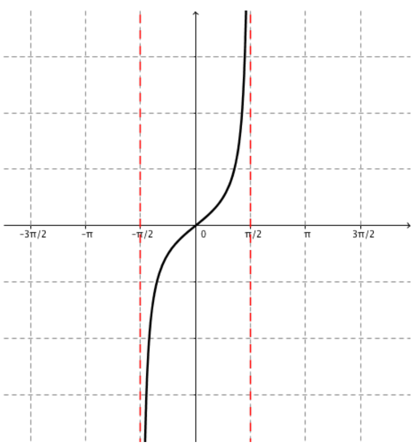
2. $\tan(\arcsin(1/3))$

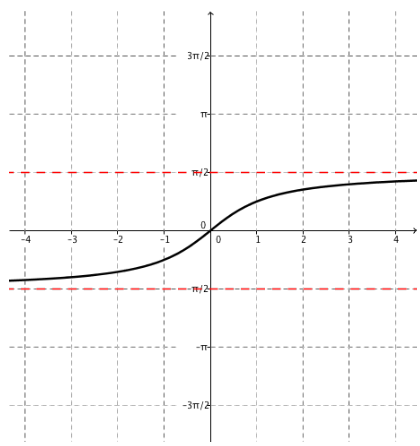




Inverse Cosine Function

$$\cos^{-1} x = y \qquad \Longleftrightarrow \qquad \cos y = x \qquad \text{and} \qquad 0 \leq y \leq \pi$$





Inverse Tangent Function

$$\tan^{-1} x = y \qquad \Longleftrightarrow \qquad \tan y = x \qquad \text{and} \qquad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

Example 12.

Simplify the expression $\cos(\arctan x)$.