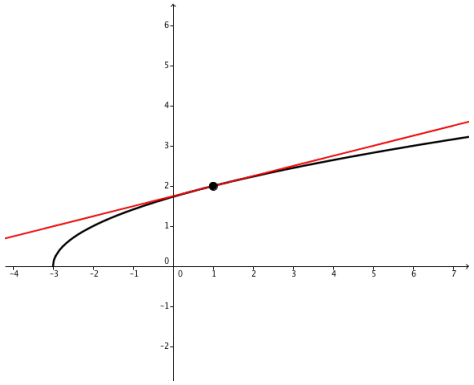
Linear Approximations

We use the tangent line at $(a, f(a))$ as an approximation to the curve $y = f(x)$ when x is near a.



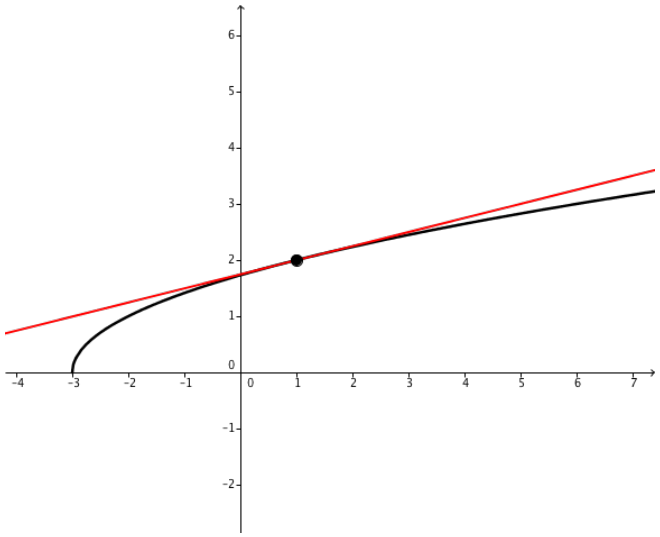
Linear Approximation

- The approximation $f(x) \approx f(a) + f'(a)(x - a)$ is called the **linear approximation** of f at a.
- We call the linear function whose graph is this tangent line the **linearization** of f at a and write

$$L(x) = f(a) + f'(a)(x - a)$$

Example 1.

Find the linearization of the function $f(x) = \sqrt{x+3}$ at $a = 1$ and use it to approximate the numbers $\sqrt{3.98}$ and $\sqrt{4.05}$. Are these approximations overestimates or underestimates?

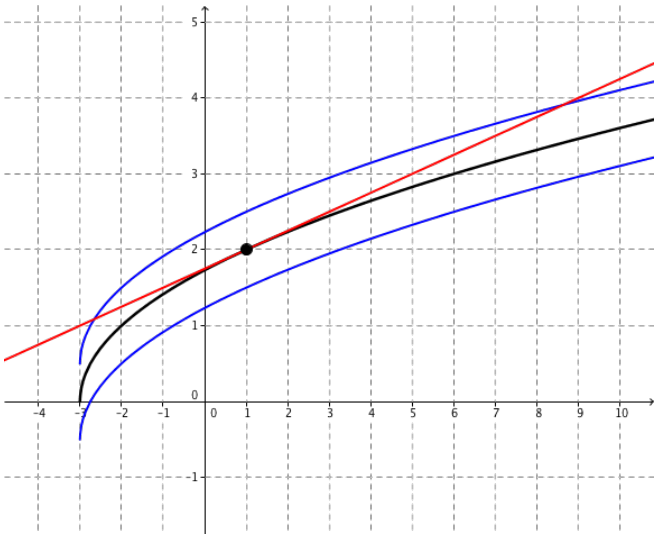


In this table we compare the actual values and estimates from the linear approximations.

	x	Estimate $L(x)$	Actual Values
$\sqrt{3.9}$	0.9	1.975	1.97484176...
$\sqrt{3.98}$	0.98	1.995	1.99499373...
$\sqrt{4}$	1	2	2.00000000...
$\sqrt{4.05}$	1.05	2.0125	2.01246117...
$\sqrt{4.1}$	1.1	2.025	2.02484567...
$\sqrt{5}$	2	2.25	2.23606797...
$\sqrt{6}$	5	2.5	2.44948974...

Example 2.

For what values of x is the linear approximation from example 1 accurate to within 0.5?



Example 3.

Find the linearization of the function $f(x) = \sin(x)$ at $a = 0$.