Maximum and Minimum Values

Absolute Maximum and Minimum

Let c be a number in the domain of f.

Absolute Maximum The value $f(c)$ such that $f(c) \geq f(x)$ for all x in the domain of f.

Absolute Minimum The value $f(c)$ such that $f(c) \leq f(x)$ for all x in the domain of f.

Absolute Extremum An absolute maximum or absolute minimum.

Local Maximum and Minimum

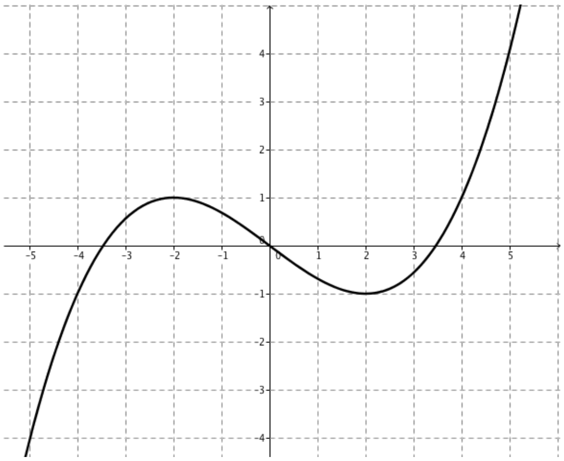
Let c be a number in the domain of f.

Local Maximum The value $f(c)$ such that $f(c) \geq f(x)$ for all x near c.

Local Minimum The value $f(c)$ such that $f(c) \leq f(x)$ for all x near c.

Local Extremum A local maximum or local minimum.

Example 1.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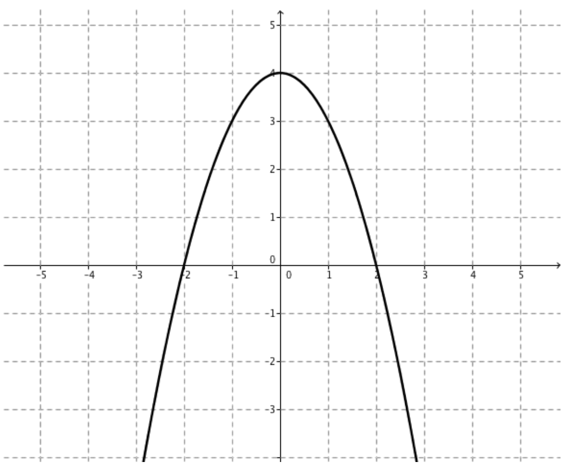
4 3 2 1 -1 -2 -3 -4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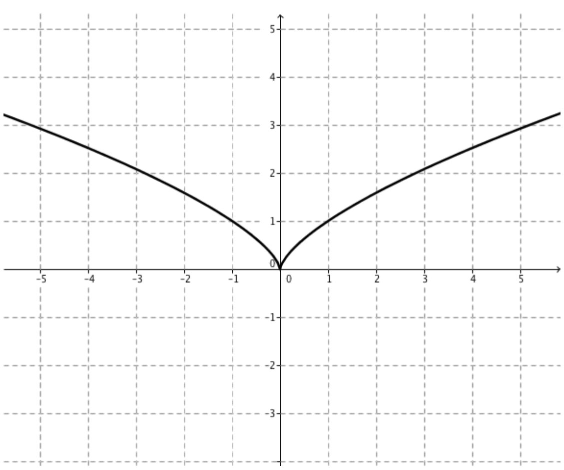
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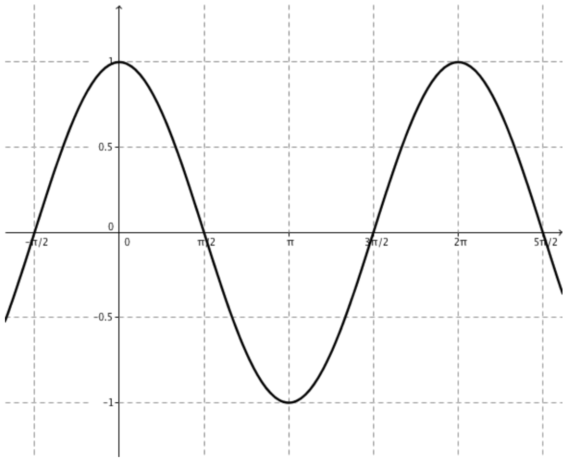
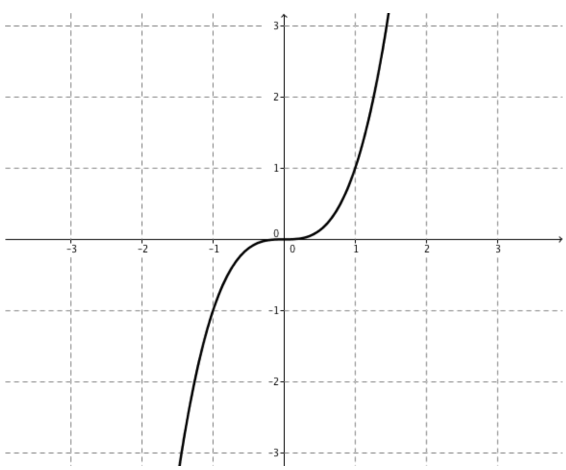
5 4 3 2 1 0 -1 -2 -3

Example 2.





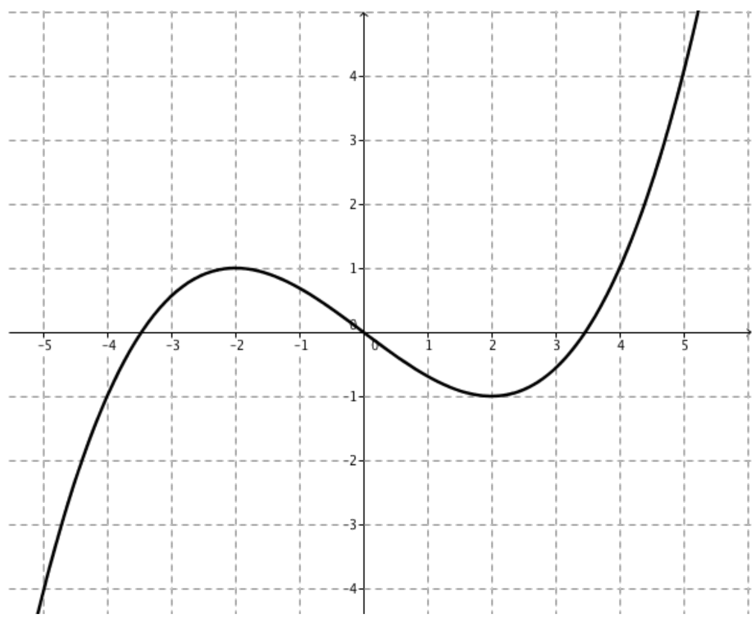
Extreme Value Theorem

A function f that is continuous on a closed interval $[a, b]$ has both an absolute maximum and an absolute minimum on that interval.

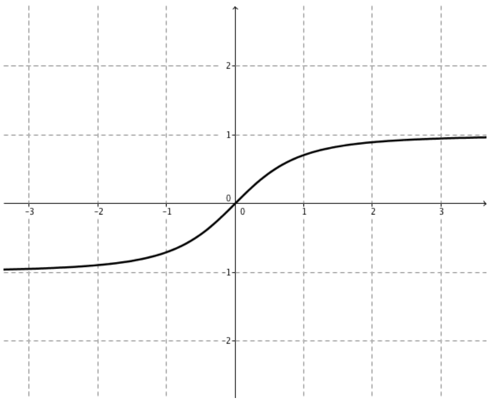
Example 3.

The graph of a function is given below. Find the absolute extrema on the intervals

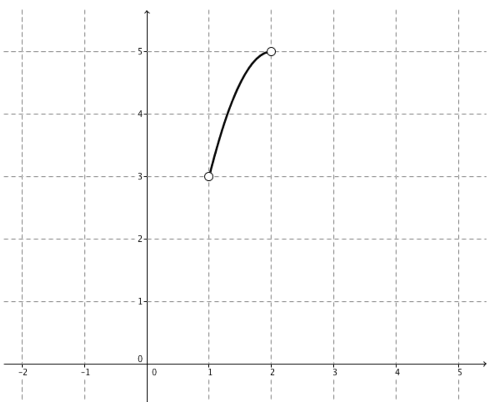
$$[-5, 5] \qquad [-3, 3] \qquad [-3, 0] \qquad [-4, 4]$$



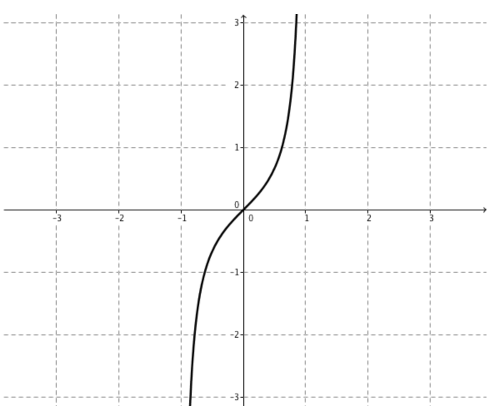
Example 4.



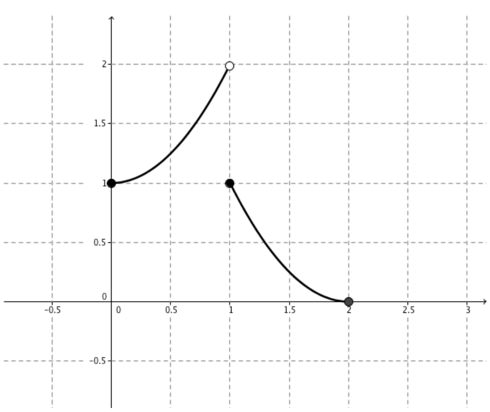
2
1
0
-3 -2 -1 0 1 2 3
-1
-2



5
4
3
2
1
0
-2 -1 0 1 2 3 4 5



3
2
1
0
-3 -2 -1 0 1 2 3
-1
-2
-3



2
1.5
1
0.5
0
-0.5 0 0.5 1 1.5 2 2.5 3
-0.5

Partition Numbers of f'

A **partition number** of the function f' is a real number c such that either

1. $f'(c) = 0$ or
2. $f'(c)$ DNE

Critical Number of f

A **critical number** of a function f is a number c in the domain of f such that either

- $f'(c) = 0$ or
- $f'(c)$ does not exist.

Example 5.

Find the critical numbers of $f(x) = x^{3/5}(4 - x)$.

How Can We Find The Local Extrema?

Fermat's Theorem

If f has a local maximum or minimum at c, then c is a critical number of f.

How Can We Find The Absolute Extrema?

Locating Absolute Extrema

Absolute extrema (if they exist) must occur at critical numbers or at endpoints.

Closed Interval Method

1. Determine if $f(x)$ is continuous on $[a, b]$.
2. Find the critical numbers of $f(x)$ that are in the interval (a, b).
3. Evaluate f at the endpoints a and b and at the critical numbers found in step 2.
   (a) The absolute maximum of f on $[a, b]$ is the largest value.
   (b) The absolute minimum of f on $[a, b]$ is the smallest value.

Example 6.

Find the absolute maximum and absolute minimum of

$$f(x) = x^3 + 3x^2 - 9x - 7$$

on each of the following intervals:

1. $[-6, 4]$
2. $[-4, 2]$
3. $[-2, 2]$

Example 7.

Find the absolute maximum and absolute minimum of

$$f(x) = x - 2\sin x \qquad\qquad 0 \leq x \leq 2\pi$$