Optimization Problems

Optimization problems are problems that involve finding the absolute maximum or the absolute minimum of a function often subject to some additional constraints.

Strategy for Solving Optimization Problems

1. Draw a diagram (if needed).
2. Introduce variables and look for relationships among them.
   (a) Define the **objective equation** - the value you want to minimize or maximize.
   (b) Define the **constraint equation** - the additional information that needs to be included.
3. Eliminate one of the variables by
   (a) Solving the constraint equation for one of the variables.
   (b) Substituting the result into the objective equation.
4. Find and verify the absolute maximum and/or minimum of the resulting function.

First Derivative Test for Absolute Extreme Values

Suppose that c is the only critical number of a continuous function f defined on an interval.

- If $f'(x) > 0$ for all $x < c$ and $f'(x) < 0$ for all $x > c$, then $f(c)$ is the absolute maximum value of f.
- If $f'(x) < 0$ for all $x < c$ and $f'(x) > 0$ for all $x > c$, then $f(c)$ is the absolute minimum value of f.

Second Derivative Test for Absolute Extreme Values

Suppose that c is the only critical number of a continuous function f defined on an interval and $f'(c) = 0$.

- If $f''(c) < 0$ then $f(c)$ is the absolute maximum value of f.
- If $f''(x) > 0$ then $f(c)$ is the absolute minimum value of f.

Example 1. [Maximizing Area]

A farmer has 2400ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

Example 2. [Minimizing Surface Area]

A cylindrical can is to be made to hold 1L of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

Example 3. [Minimizing Distance]

Find the point on the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$.