

The Definition of Derivative (as a Function)

In the previous section we considered the derivative of a function f at a fixed number a

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

If we wanted to compute the derivative for multiple a values, say $a = 0$, $a = 1$, $a = 2$, computing the limit multiple times would be tedious.

It is usually helpful to compute this limit where we let $a = x$. In this way we obtain an expression for f' that we can simply evaluate to find the derivative.

Derivative

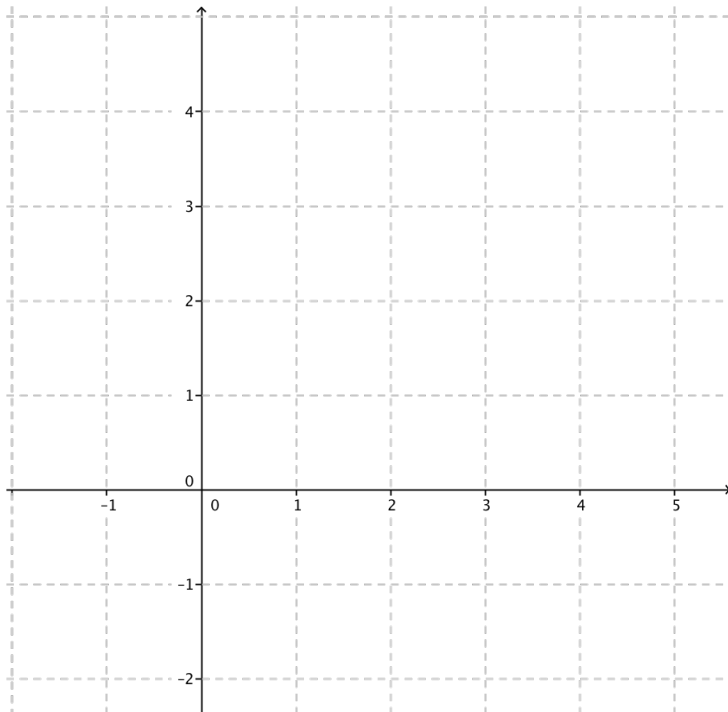
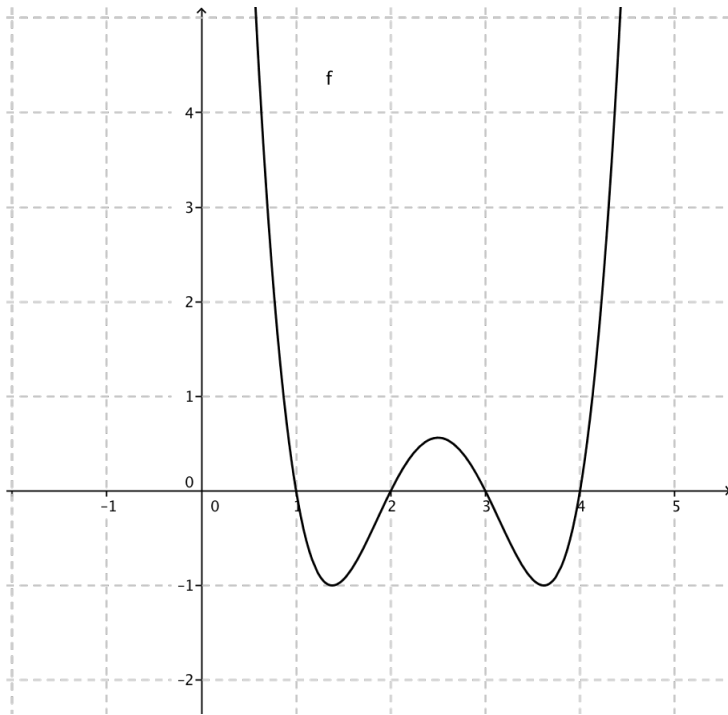
The derivative of a function f is

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

where this limit exists.

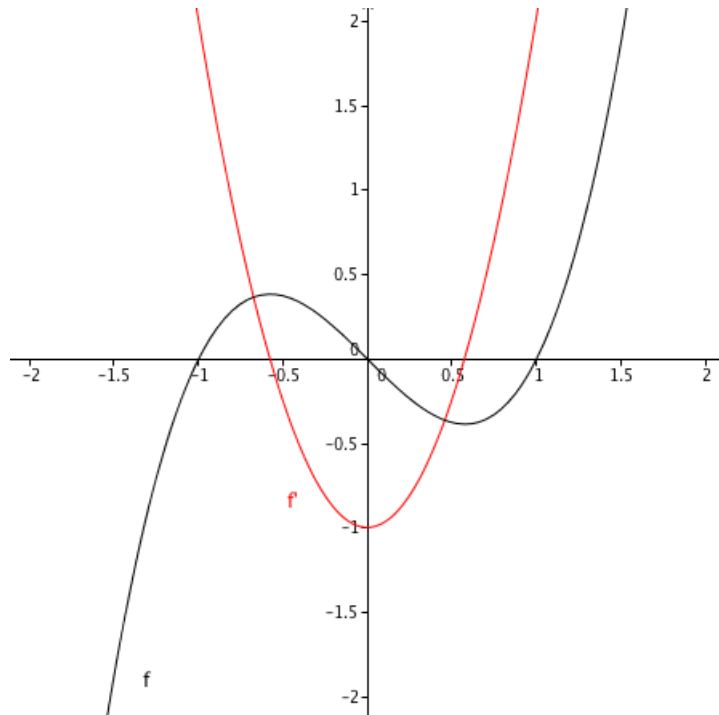
Example 1.

The graph of a function f is given below. Use it to sketch the graph of the derivative f' .



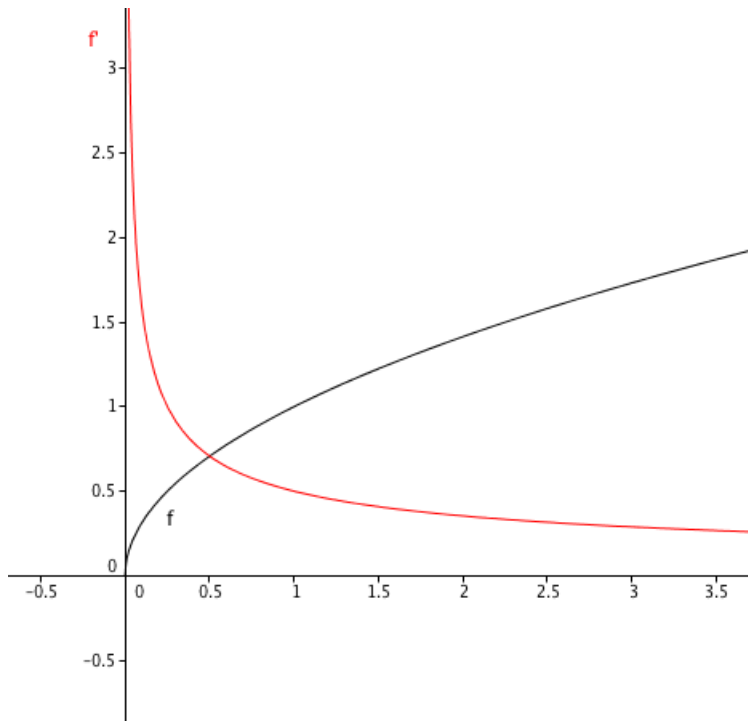
Example 2.

If $f(x) = x^3 - x$, find a formula for $f'(x)$. Compare the graphs of f and f' .



Example 3.

If $f(x) = \sqrt{x}$, find the derivative of f . State the domain of f' .



Example 4

Find f' if $f(x) = \frac{1-x}{2+x}$.

Notation

Differentiable

- A function f is **differentiable at a** if $f'(a)$ exists.
- It is **differentiable on an open interval** (a, b) [or (a, ∞) , $(-\infty, a)$, or $(-\infty, \infty)$] if it is differentiable at every number in the interval.

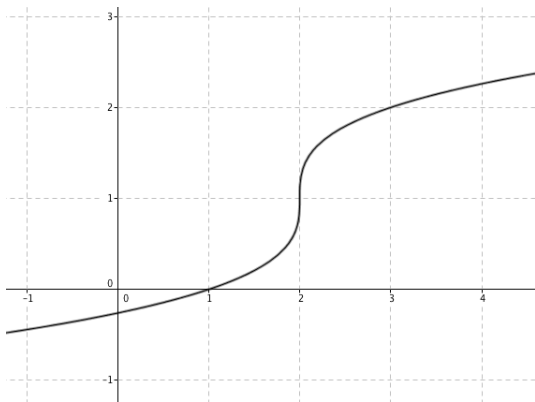
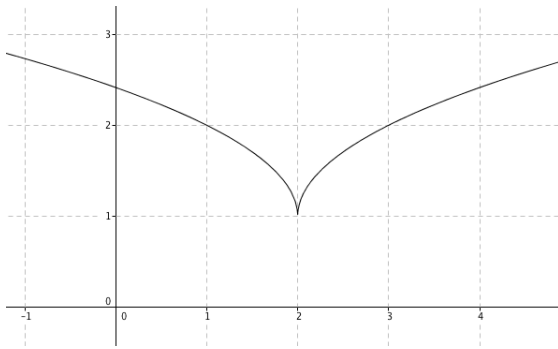
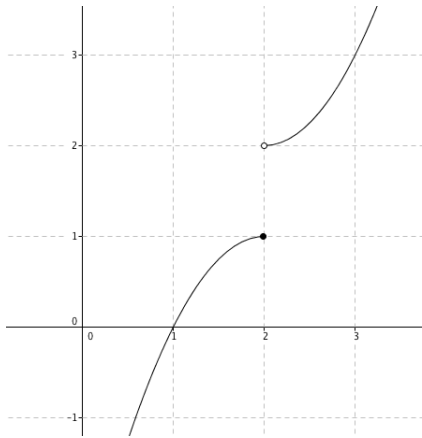
Example 5

Where is the function $f(x) = |x|$ differentiable?

Theorem

If f is differentiable at a , then f is continuous at a .

Examples of Non-Differentiability



Higher Derivatives

Interpretation of Higher Derivatives

Example 6

If $f(x) = x^3 - x$, find $f''(x)$, $f'''(x)$, and $f^{(4)}(x)$.