The Definition of Derivative (at a Point)

The Tangent Line

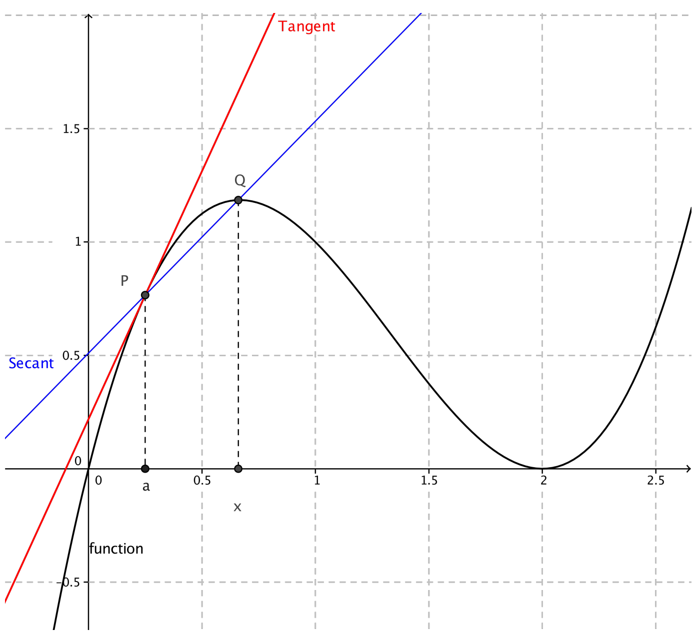
Recall that we estimated the slope of the tangent line to a curve at a point by considering the slopes of secant lines near that point. We now have the machinery to better define this limiting process.

Tangent Line

The **tangent line** to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through P with slope

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

provided that this limit exists.



The equation of a line is

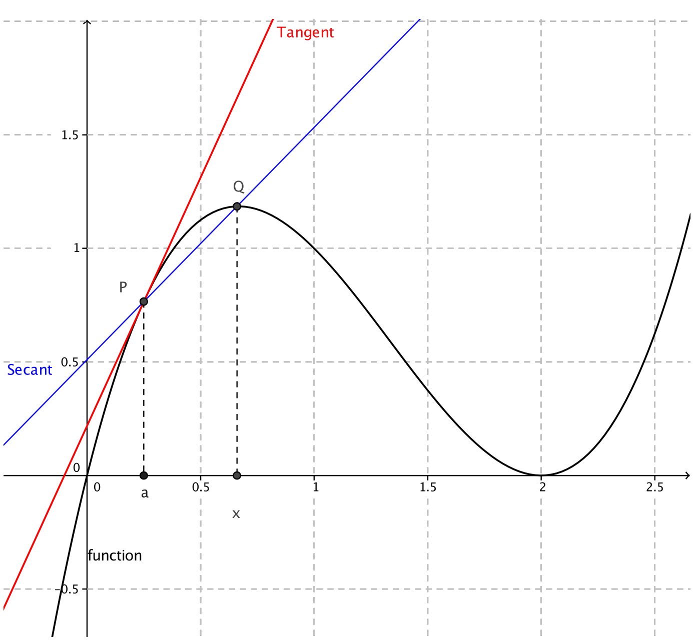
$$(y - y_0) = m\,(x - x_0)$$

Example 1.

Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.

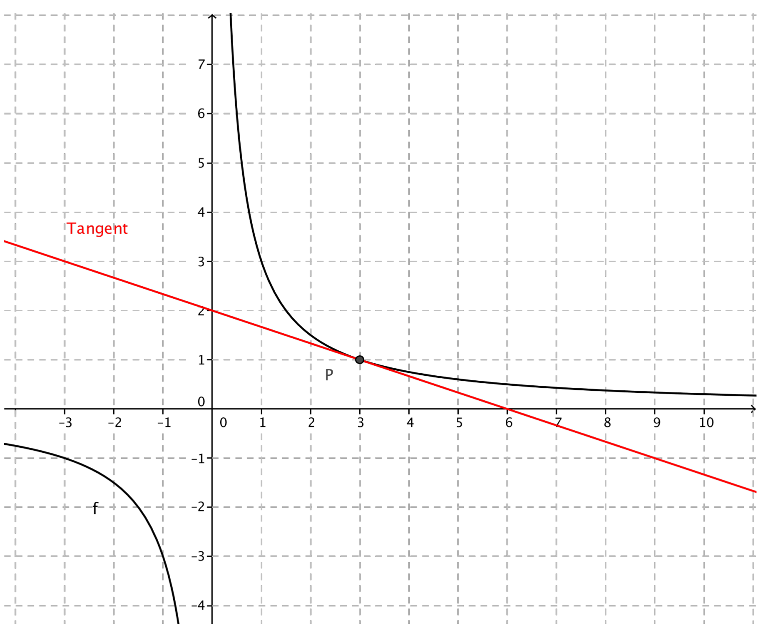
Alternative Slope of Tangent Line

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$



Example 2.

Find an equation of the tangent line to the hyperbola $y = 3/x$ at the point $(3, 1)$.



Instantaneous Velocity

Suppose an object moves along a straight line according to an equation of motion $s = f(t)$, where s is the displacement of the object from the origin at time t.

- $s = f(t)$ is called the **position function**

- **average velocity**

- **velocity** (instantaneous velocity)

- **speed**

Example 3

Suppose that a ball is dropped from a height of $450m$ above the ground.

1. Find the velocity of the ball after 5 *sec*.
2. Find the velocity of the ball when it hits the ground.

The Definition of Derivative

Derivative of f at a

The **derivative of a function f at a number a**, denoted by $f'(a)$, is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

if this limit exists.

Example 4

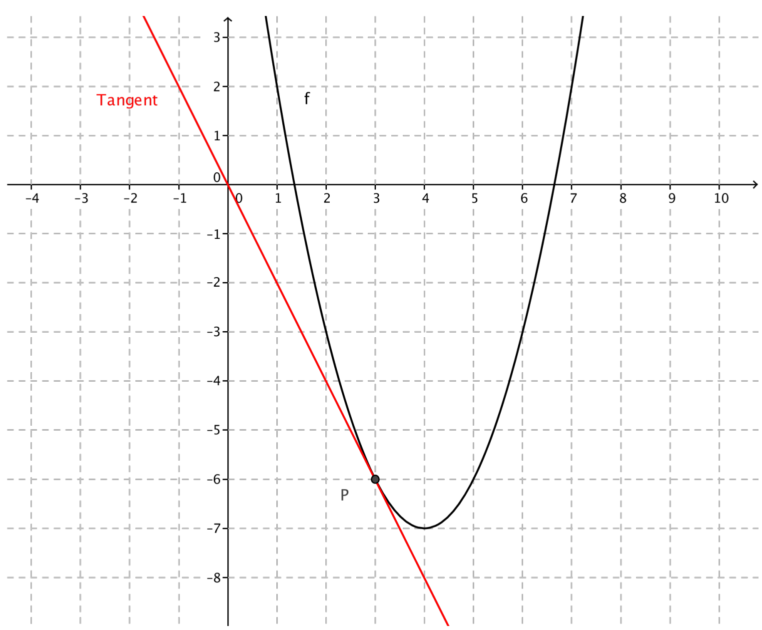
Find the derivative of the function $f(x) = x^2 - 8x + 9$ at the number a.

Tangent Line

The **tangent line** to $y = f(x)$ at $(a, f(a))$ is the line through $(a, f(a))$ whose slope is equal to $f'(a)$.

Example 5

Find an equation of the tangent line to parabola $f(x) = x^2 - 8x + 9$ at the point where $a = 3$.



Rates of Change

We can generalize the preceeding discussion to the case where y is any quantity that depends on another quantity x.

- $y = f(x)$ is a function that depends on the context of the problem

- **average rate of change of y with respect to x** over the interval $[x_1, x_2]$

- **instantaneous rate of change of y with respect to x** at $x = x_1$