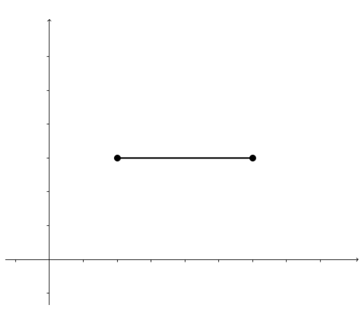
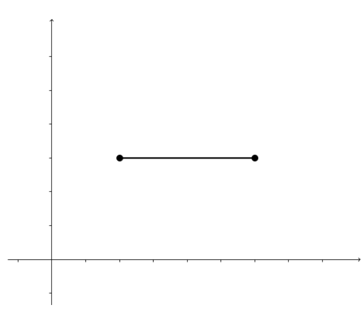
The Mean Value Theorem

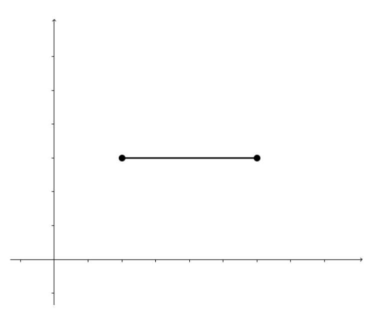
Rolle's Theorem

- Let f be a function such that:
  1. f is continuous on the closed interval $[a, b]$.
  2. f is differentiable on the open interval (a, b).
  3. $f(a) = f(b)$

  - Then there is a number c in (a, b) such that $f'(c) = 0$.







Example 1.

Let's apply Rolle's Theorem to the position function $s = f(t)$ of an object moving in a straight line.

Intermediate Value Theorem

- Let f be a function such that:
  1. f is continuous on the closed interval $[a, b]$.
  2. $f(a) \neq f(b)$
- If N is any number between $f(a)$ and $f(b)$ then there exists a number c in (a, b) such that $f(c) = N$.

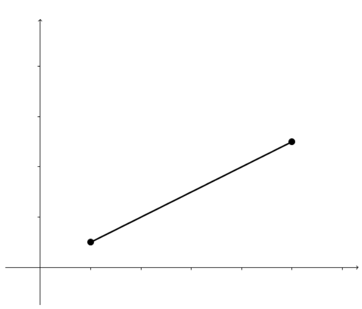
Example 2.

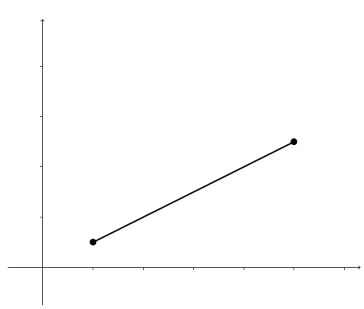
Prove that the equation $x^3 + x - 1 = 0$ has exactly one real root.

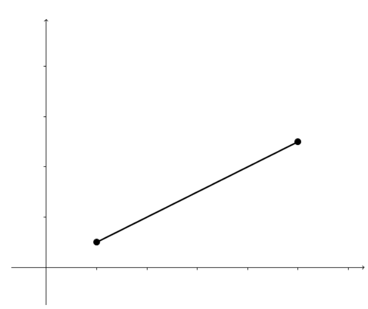
The Mean Value Theorem

- Let f be a function such that:
  1. f is continuous on the closed interval $[a, b]$.
  2. f is differentiable on the open interval (a, b).

  - Then there is a number c in (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$







Example 3.

Verify the Mean Value Theorem for the function $f(x) = x^3 - x$ on the interval $[0, 2]$.

Example 4.

Interpret the result of the Mean Value Theorem when $s = f(t)$ is a position function for an object moving in a straight line.

Example 5.

Suppose that $f(0) = -3$ and $f'(x) \leq 5$ for all values of x. How large can $f(2)$ possibly be?

Theorem

If $f'(x) = 0$ for all x in an interval (a, b), then f is constant on (a, b).

Corollary

- If $f'(x) = g'(x)$ for all x in an interval (a, b),
  - Then $f - g$ is constant on (a, b); that is, $f(x) = g(x) + C$ where C is a constant.

Example 6.

Prove the identity $\tan^{-1} x + \cot^{-1} x = \pi/2$.