The Precise Definition of a Limit at Infinity

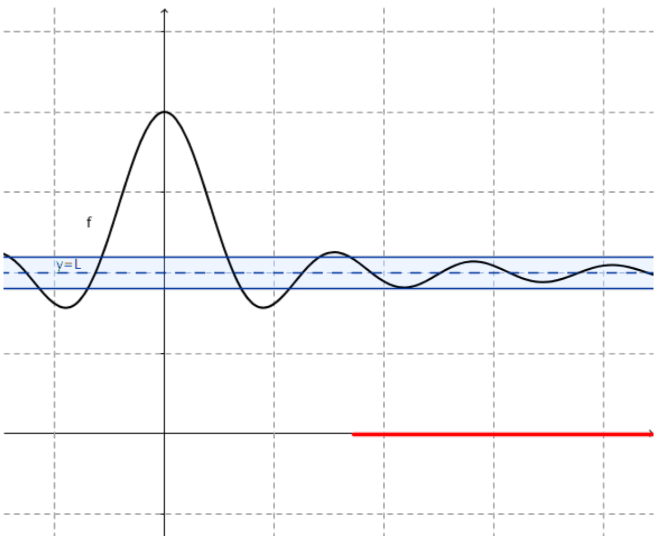
Limit at + Infinity

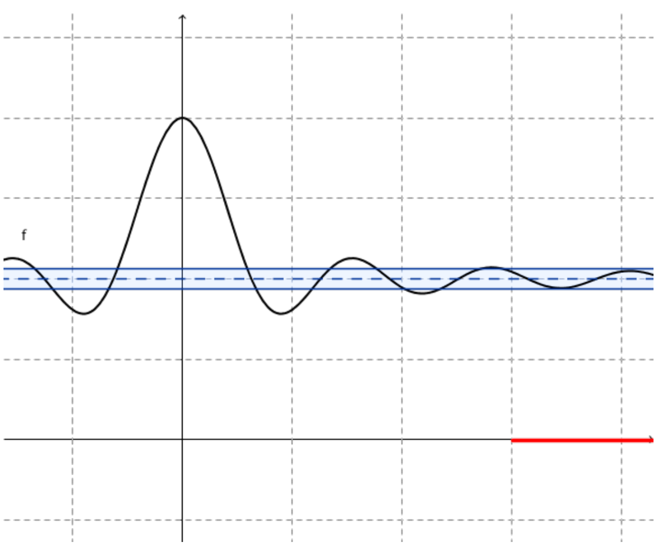
Let f be a function defined on some interval (a, ∞). Then

$$\lim_{x \to \infty} f(x) = L$$

means that for every $\epsilon > 0$ there is a corresponding number $N > 0$ such that

$$\text{if} \qquad x > N \qquad \text{then} \qquad |f(x) - L| < \epsilon$$





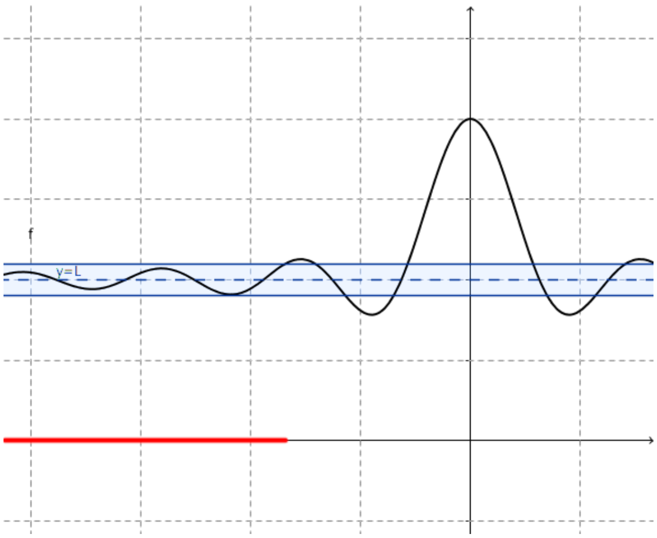
Limit at - Infinity

Let f be a function defined on some interval $(-\infty, a)$. Then

$$\lim_{x \to -\infty} f(x) = L$$

means that for every $\epsilon > 0$ there is a corresponding number $N < 0$ such that

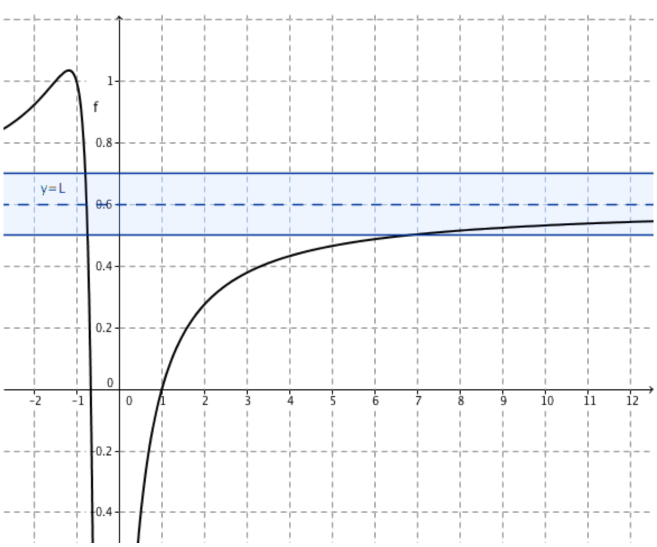
$$\text{if} \qquad x < N \qquad \text{then} \qquad |f(x) - L| < \epsilon$$



Example 1.

Use the graph to find a number N such that

$$\text{if} \qquad x > N \qquad \text{then} \qquad \left| \frac{3x^2 - x - 2}{5x^2 + 4x + 1} - \frac{3}{5} \right| < 0.1$$



Example 2.

Prove that $\lim_{x \to \infty} \frac{1}{x} = 0$.

Infinite Limit at + Infinity

Let f be a function defined on some interval (a, ∞). Then

$$\lim_{x \to \infty} f(x) = \infty$$

means that for every positive number M there is a corresponding number $N > 0$ such that

$$\text{if} \qquad x > N \qquad \text{then} \qquad f(x) > M$$

Example 3.

Prove that $\lim_{x \to \infty} x^2 = \infty$.