

The Product and Quotient Rules

Introduction

In this section we determine how to calculate the derivative of the product and quotient of two functions in terms of the original functions and their derivatives.

Product Rule

The Product Rule

If f and g are both differentiable, then

$$\frac{d}{dx} [f(x)g(x)] = f(x)\frac{d}{dx} [g(x)] + g(x)\frac{d}{dx} [f(x)]$$

Example 1.

For $f(x) = xe^x$

1. Find $f'(x)$.

2. Find $f^{(n)}(x)$.

Example 2.

Differentiate the function $f(t) = \sqrt{t}(a + bt)$.

Example 3.

If $f(x) = \sqrt{x}g(x)$, where $g(4) = 2$ and $g'(4) = 3$, find $f'(4)$.

Quotient Rule

The Quotient Rule

If f and g are differentiable, then

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2}$$

Example 4.

Let $y = \frac{x^2 + x - 2}{x^3 + 6}$. Find y' .

Example 5.

Find an equation of the tangent line to the curve $y = \frac{e^x}{1+x^2}$ at the point where $x = 1$.

Example 6.

Find the derivative of $f(x) = \frac{3x^2 + 2\sqrt{x}}{x}$.