

The Substitution Rule for Definite Integrals

Method 1: Find the Antiderivative First

1. Find the antiderivative of the function using the substitution rule.
2. Evaluate the antiderivative as usual.

Example 1a.

Evaluate $\int_0^5 \frac{1}{x^2 + 10} (2x) dx$

The Substitution Rule for Definite Integral

If g' is continuous on $[a, b]$ and f is continuous on the range of $u = g(x)$, then

$$\int_a^b f(g(x))g'(x) dx = \int_{g(a)}^{g(b)} f(u) du$$

Method 2: Change the Limits of Integration First

1. Define the u substitution.
2. Eliminate the original variable by expressing the integral in terms of u and du .
3. **Change the limits of integration in terms of u .**
4. Evaluate the new integral.

Example 1b.

Evaluate $\int_0^5 \frac{1}{x^2 + 10} (2x) dx$

Example 2.

1. Evaluate $\int_0^4 \frac{1}{\sqrt{2x+1}} dx$

2. Evaluate $\int_1^2 \frac{1}{(3-5x)^2} dx$

Example 3.

1. Evaluate $\int_1^e \frac{\ln x}{x} dx$

2. $\int_0^4 \frac{x}{\sqrt{2x+1}} dx$