

The Substitution Rule

Introduction

Differentials

If $u = g(x)$ is a differentiable function, then the **differential** du is

$$du = g'(x)dx$$

Example 1. [Differentials]

1. $u = x^2$

2. $u = \ln(4 + 5t)$

Integration by Substitution

The Substitution Rule

If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I , then

$$\int f(g(x))g'(x) dx = \int f(u) du$$

Procedure: Integration by Substitution

1. Define the u substitution.
2. Eliminate the original variable by expressing the integral in terms of u and du .
3. Evaluate the new integral.
4. Convert this antiderivative back to the original variable.

Example 2.

1. $\int (3x + 4)^{10}(3) dx$

2. $\int (2x)e^{x^2} dx$

3. $\int \frac{3x^2}{1+x^3} dx$

Differential du off by a Constant

Example 3.

1. $\int te^{-t^2} dt$

2. $\int x^3 \cos(x^4 + 2) dx$

3. $\int \sqrt{2t + 1} dt$

4. $\int \frac{x}{\sqrt{1 - 4x^2}} dx$

Differential du off by More Than a Constant

Example 4.

1. Find $\int \frac{x}{\sqrt{x+2}} dx$

2. Find $\int x^5 \sqrt{1+x^2} dx$

Example 5.

Find $\int \tan x dx$