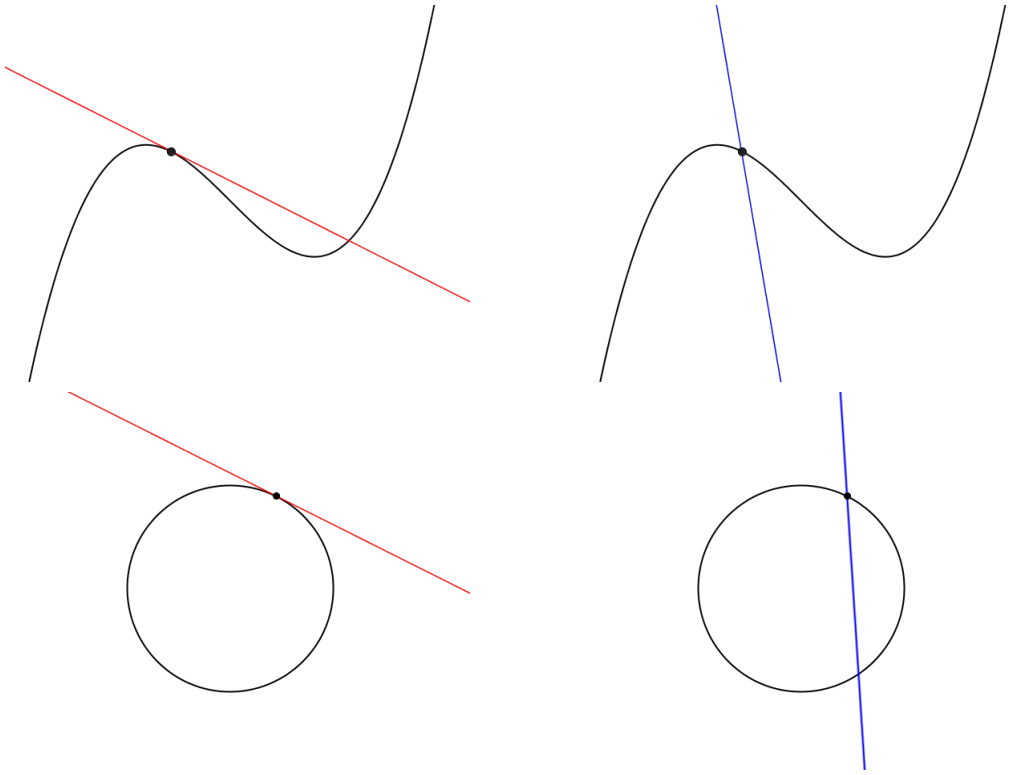
The Tangent and Velocity Problems

Limits are central to our study of calculus. In this lecture we introduce two problems that motivate our study of limits and derivatives.

The Tangent Problem.

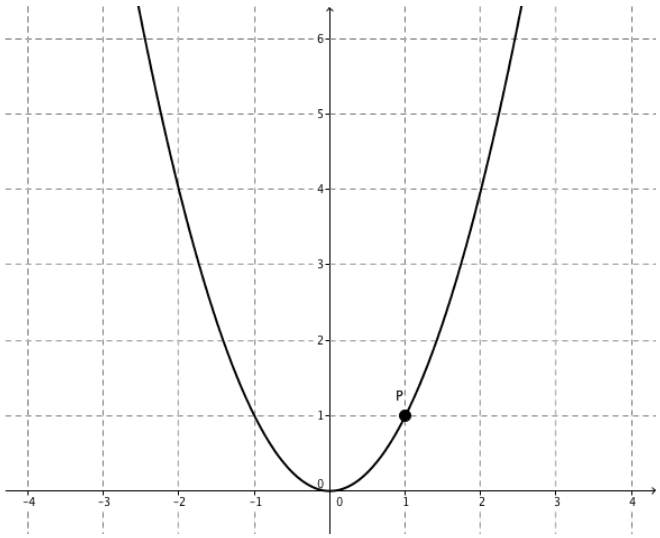
Here we try to find the tangent to a curve. The difficulty arises in precisely defining what we mean by 'tangent to a curve'. What are some properties we would like the tangent line to possess?



Example 1.

Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.

Idea: Approximate the slope of the tangent line by calculating the slope of the **secant line** PQ for a point Q sufficiently close to point P on the curve.



Next we compute the slopes m_{PQ} as the point Q gets closer to the point P along the curve.

$x < 1$	m_{PQ}
0	1
0.5	1.5
0.9	1.9
0.99	1.99
0.999	1.999

$x > 1$	m_{PQ}
2	3
1.5	2.5
1.1	2.1
1.01	2.01
1.001	2.001

The Velocity Problem

We have an intuitive understanding that an object in motion has a velocity defined for each moment. To be specific we call this the **instantaneous velocity**. But how do we define the instantaneous velocity? We will explore this further by considering the case of an object moving in a straight line.

Example 2

Suppose that a ball is dropped from a height of $450m$ above the ground. Find the velocity of the ball after 5 seconds.