MTH 418/518 Provisional Syllabus

Spring 2017

Ch 1 The heat equation

1.2 How heat flows in a slender rod

1.3 Boundary conditions

1.4 Equilibrium temperature distribution

1.5 How heat flows in multidimensional objects

Ch 2 The method of separation of variables

2.3 Boundary conditions

The method

Time-dependence

Space-dependence

Product solutions and superpositions of them

Orthogonality of sines and solution of the initial value problem

Examples

2.4 Insulated ends

Heat flow in a ring

2.5 Laplace's equation

On a rectangle and on a disk

Qualitative properties of solutions

Ch 3 Fourier series

3.2 Conditions for convergence

3.3 Sine and cosine series

The Gibbs phenomenon

3.4 Term-by-term differentiation

Ch 4 The wave equation

4.2 Derivation for a stretched string

4.3 Boundary conditions

4.4 Vibrations of a string with fixed ends, normal modes

Project 1: Fourier analysis of a recording of a piano string

Vibration of 2D membranes considered later in Ch 7

Vibration of a bar

The equation of motion

Boundary conditions

Modes of vibration

Ch 5 Sturm-Liouville eigenvalue problems

5.2 Examples: heat flow in nonuniform rod, axisymmetric heat flow

5.3 Definition and theorems, Rayleigh quotient

5.4 Heat flow in a nonuniform rod (no sources)

5.5 Proofs of some of the parts of the Sturm-Liouville theorem

5.6 The Rayleigh quotient

5.7 Vibrations of a nonuniform string

Project 2: Heat flow in a particular nonuniform rod

5.8 Boundary conditions of the third kind

5.9 Approximation of large eigenvalues

5.10 Approximation of solutions by truncated generalized Fourier series

Ch 7 Higher-dimensional PDEs

7.2 Vibrating membrane (drum-head), 2D heat equation

7.3 Vibrating rectangular membrane

7.4 Sturm-Liouville-type theorems for Helmholtz equation

7.5 Green's formula, self-adjoint operators

7.7 Vibrating circular membrane and Bessel functions

7.7.2 Separation of variables

7.7.3 Eigenvalue problems

7.7.4-7 Bessel's ODE

7.7.8-9 Solution of initial value problem

Project 3: Modes of a cymbal (?)

Ch 8 Nonhomogeneous problems

8.2 Heat flow with sources and nonhomogeneous boundary conditions

8.3 The method of eigenfunction expansion (homogeneous boundary conditions)

Ch 9 Green's functions

9.3 Green's functions for boundary value problems for ODEs

9.5 Green's functions for Poisson's equation

Ch 10 Infinite-domain problems - the Fourier transform method

10.2 Heat equation on an infinite domain

10.3 The Fourier transform pair

10.4 Fourier transform solution of the heat equation

10.5 Heat flow on semi-infinite domains: Fourier sine and cosine transforms

10.6 Worked examples using transforms

Ch 12 Method of characteristics

12.2 First order wave equation

12.3 1D wave equation