Homework #7

due 11:59pm Sunday April 17

1.

Regarding the Galerkin problem of Homework 6 #2, we have a theorem that says the error in the Galerkin approximation in the norm $||e||_1$ given by $||e||_1^2 = \int (e')^2 + e^2$ goes to zero as $1/N^2$. And yet you can see in my Homework 6 solutions that the maximum of e' goes to zero slower than that - only as $1/N$. So if we chose the norm $\max(|e'| + |e|)$ instead, we could only claim an error of order $1/N$.

a.

What's going on? Aren't all norms equivalent? (Hint: no.) Two norms $|| \cdot ||_a$ and $|| \cdot ||_b$ are said to be equivalent if $c||x||_a \leq ||x||_b \leq C||x||_a$ for all x for some c, C.

b.

Construct a simple family of functions in $C[0, 1]$ parametrized by N such that their L^1 norm goes to zero as N goes to infinity, while their L^∞ (or max or uniform) norm does not.

2.

Exercise 1-2 on p13 of Ames book (see your email of April 13).