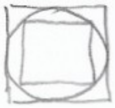
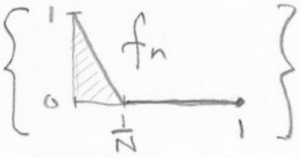
Homework #7

1 a. No. Only in finite-dimensional spaces are all norms equivalent.



b. $\left\{ \right.$



$\left. \right\}$

$$\lim_{N\to\infty} \int_0^1 |f_n| = 0$$

$$\max_{[0,1]} f_n = 1 \quad \forall n$$

for HW7

Ames 1-3 ③

2. Example 2 1D isentropic flow of perfect gas

$u_t + uu_x + \frac{1}{\rho} p_x = 0$ momentum balance

$\rho_t + \rho u_x + u\rho_x = 0$ continuity

u = velocity

p = pressure

ρ = density

c = vel. of sound

γ = ratio of specific heats

$$p\rho^{-\gamma} = \alpha \text{ const} \quad , \quad c^2 = \frac{dp}{d\rho}$$

Eliminate the pressure & write 2 eqns for u & ρ.

No: If $\frac{dp}{d\rho} = \underset{\nwarrow \text{ not const.}}{c^2}$, then $\boxed{p = c^2\rho} + \kappa$. But $p|_{\rho=0}$ must $= 0$, so $\kappa = 0$.

$p = \alpha\rho^\gamma$, c is a local speed.

$$u_t + uu_x + \frac{1}{\rho}\alpha\gamma\rho^{\gamma-1}\rho_x = \boxed{\begin{aligned} &u_t + uu_x + \alpha\gamma\rho^{\gamma-2}\rho_x = 0 \\ &\rho_t + \rho u_x + u\rho_x = 0 \end{aligned}}$$

$$A = \begin{array}{c} \quad u \qquad\qquad \rho \\ \left[\begin{array}{cccc} \overset{a_1}{1} & \overset{b_1}{u} & \overset{c_1}{0} & \overset{d_1}{\alpha\gamma\rho^{\gamma-2}} \\ \overset{a_2}{0} & \overset{b_2}{\rho} & \overset{c_2}{1} & \overset{d_2}{u} \\ 1 & y' & 0 & 0 \\ 0 & 0 & 1 & y' \end{array}\right] \end{array}$$

Typos: y' should be x'.

$$\text{discr} A = (1\cdot u - 0\cdot(\ ) + u\cdot 1 - \rho\cdot 0)^2 - 4(1\cdot 1 - 0\cdot(\ ))(u\cdot u - \rho\alpha\gamma\rho^{\gamma-2}) = 4u^2 - 4(u^2 - \alpha\gamma\rho^{\gamma-1})$$

Oops, chopped off the end.
discr A = alpha gamma rho^(gamma-1). Since alpha, gamma, and rho are all positive quantities, the discriminant is positive, we have two real roots, and the equation is hyperbolic.