Homework #8

due 11:59pm Sunday May 1

1.

(a) Find a formula for the eigenvalues of the Dufort-Frankel scheme applied to the heat equation $u_t = u_{xx}$, in terms of $\sigma = k/h^2$ and α. (The eigenfunctions are $e^{\hat{i}\alpha i}\lambda^j$.)

(b) Demonstrate stabiltity by showing that for all $\sigma > 0$ both eigenvalues have modulus ≤ 1 and that any eigenvalue of modulus 1 has multiplicity 1. Divide the work into the following cases: $\alpha = 0$, $\alpha = \pi$, $\lambda_\pm$ complex, and $\lambda_\pm$ real with α in $(0, \pi)$.

Hints: If $\lambda_\pm$ are complex, then $|\lambda_+|^2 = |\lambda_-|^2 = \lambda_+\lambda_-$; eigenvalues are continuous in the coefficients of the characteristic equation, so showing that $\lambda_\pm = \pm 1$ is impossible would be useful.

(c) Show that the schemes is consistent with the heat equation only if the time step k goes to zero faster than the spatial grid spacing h, and say what PDE the scheme is consistent with if $k = 2h \to 0$.

2.

Use your Poisson code to determine the temperature at the center of the square plate in the BVP we discussed in class. Show some evidence of convergence of this estimate as the grid is refined.